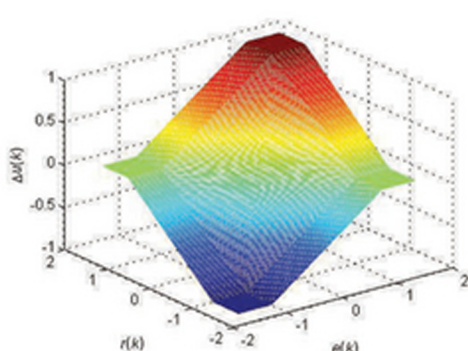
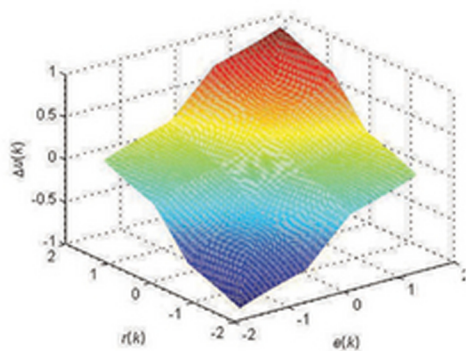




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David B. Fogel, Series Editor

# INTRODUCTION TO TYPE-2 FUZZY LOGIC CONTROL

## THEORY AND APPLICATIONS



JERRY M. MENDEL, HANI HAGRAS, WOEI-WAN TAN,  
WILLIAM W. MELEK, HAO YING



# **INTRODUCTION TO TYPE-2 FUZZY LOGIC CONTROL**

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*To*  
*the memory of*  
*Ebrahim Mamdani (1943–2010)*  
*Founder of Fuzzy Logic Control*



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## PREFACE

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When Lotfi Zadeh invented fuzzy sets in 1965, he never dreamt that the field in which they would be most widely used would arguably be the one that became the most hostile to the concept of fuzziness, namely control. Perhaps this was because the word “fuzzy” in Western civilization does not have a positive connotation and suggests an abandonment of mathematical rigor, one of the cornerstones of control. Perhaps it was because some famous mathematical probabilists (incorrectly) claimed that there was no difference between a fuzzy set and subjective probability. Perhaps it was because for almost a decade, until the 1974 seminal paper by Prof. Ebrahim Mamdani, who founded the field of fuzzy logic control and to whose memory our book is dedicated, there were no substantial real-world applications for fuzzy sets. Or, perhaps, it was because after the founding of this field many exaggerated claims were made by the fuzzy logic control community that flew in the face of mathematical rigor and did not pay attention to the same metrics that were and still are the cornerstones for control and cannot be ignored.

Now, 40 years after Mamdani’s seminal paper, fuzzy logic control using regular (i.e., type-1) fuzzy sets and logic has been extensively studied, applied to practical problems, and is very widely used in many real-world applications. It can and has been studied with the same level of mathematical rigor that control theorists are accustomed to, and is now considered a matured field; however, it still has some shortcomings. Its major shortcoming (in the opinions of the authors of this book) goes back to one of the earliest criticisms made about a type-1 fuzzy set, namely the unfuzziness of its membership function, that is, the word “fuzzy” has the connotation of being uncertain. But how can this connotation be captured by a membership function that is completely certain?

Importantly, in 1975 Zadeh introduced more general kinds of fuzzy sets in which their membership function grades are themselves fuzzy. The two most widely studied of these are *interval-valued fuzzy sets* and *type-2 fuzzy sets*. For the former, the membership grade is a uniformly weighted interval of values, whereas for the latter the membership grade is a nonuniformly weighted interval of values. Obviously, interval-valued fuzzy sets are a special case of type-2 fuzzy sets and are therefore called by many (as we do in this book) *interval type-2 fuzzy sets*.

Why should using type-2 fuzzy sets be of interest to the fuzzy logic control community? This question is answered in great detail in this book, but two short answers are: (1) they are more robust to system uncertainties and can provide better control system performance than type-1 fuzzy sets; and (2) there is now more than a critical